Feed or die: Space covering by a random walk and prey-predator population dynamics

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To describe the dynamics of an ecological system with several interacting species, at least three ingredients must be simultaneously considered: 1) how the number and distribution of consumers (or predators) and resources (or prey) vary in the environment, 2) how interactions between individuals take place, 3) how interactions between individuals translate into births and deaths. The classical model describing such a system is the deterministic Lotka-Volterra system (and its numerous derivations) where these three ingredients are generally implicitly assumed to occur at the same time scales. The aim of this PhD thesis is to study the properties of such systems using a stochastic approach, seeking in particular to highlight different regimes depending on the assumptions made about these three ingredients.

The PhD student will build on the work already carried out by the supervisors: a stochastic model of the interaction rates between prey and predators under the assumption that resources are instantaneously regenerated once consumed (Billiard et al. 2018), or on the contrary that resources are never regenerated during the random walk in spaceof the predator (Bansaye et al., in prep.), or indeed a multiscale approximation of population dynamics but in a non-spatial model with general interaction times (Bansaye and Cloez, 2021).

The aim of this thesis will be to study some relevant intermediate models where the time scales involved in the rates of interaction and population size variation are explicitly controlled. In particular, we will be interested in the lifetime of a predator whose survival depends on the amount of prey actually consumed as a function of the rate of prey regeneration. We will also want to determine under which hypotheses of the relationship of scales between consumption and reproduction speed, prey and predators can coexist over a long time.

The thesis project is in the mathematical fields of space covering by a random walk (Le Gall, 1986), branching random walks, stochastic processes in random environments, and birth and death processes with interaction.

Profile sought: This project is open to any student of mathematics or theoretical physics with strong skills in probability and stochastic processes. The student should also have a strong interest in interdisciplinary interactions (in ecology) since the co-supervision will be ensured by two mathematicians and a biologist.

References

Bansaye, V., & Cloez, B. (2021). From the distributions of times of interactions to preys and predators dynamical systems. A paraître à Journal of Mathematical Biology.
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Billiard, S., Bansaye, V., & Chazottes, J. R. (2018). Rejuvenating functional responses with renewal theory. Journal of The Royal Society Interface, 15(146), 20180239.
Le Gall, J-F. Propriétés d'intersection des marches aléatoires. Communications in mathematical physics, 104(3):471–507, 1986.

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